The Remarkable Theorem of Gauss

Student: Abdulrahman Alenazi
Mentor: Prakhar Gupta
Directed Reading Program
May 2021
Introduction

• Have you noticed what happens to pizzas when folding it?

• Why all 2D maps of earth are going to be distorted?
Curvature of A Curve

• A curve is a map from an interval to $\mathbb{R}^3$.
• A regular curve is a curve where its first derivative is never 0.
• The curvature is the acceleration (second derivative) of a unit speed parametrization of a curve. Length is time passed in this case.
• In case not unit-speed, we could use the following derivation:

$$\kappa = \frac{\|\dot{y} \times \ddot{y}\|}{\|\dot{y}\|^3}$$
A surface

- A surface is roughly an object embedded in $\mathbb{R}^3$ in such a way that an ant on top of it would think it is a plane.
Is sphere a surface?
Tangent space and normal
Local Isometry

- It preserves lengths of curves on a surface.
- Examples are folding a paper or a tissue.
Principal curvature and Gaussian curvature

• Gaussian Curvature is the multiplication of the two principal curvatures
• Principal curvatures are the max and min normal curvatures
Gauss Remarkable Theorem

• Local Isometries reserve the Gaussian curvature!!