



# Lie Algebras to Root Systems

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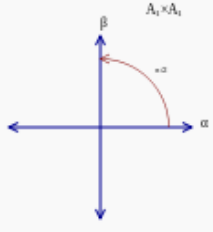
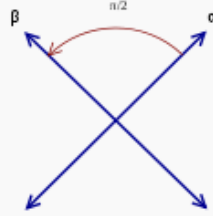


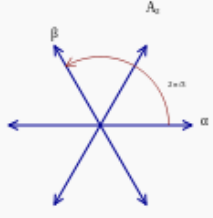
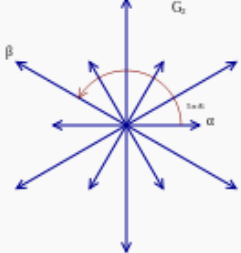


# Lie Algebras

- A **Lie algebra** is a vector space over some field with some binary operation that satisfies the following axioms:
  - Bilinearity
    - $[ax + by, z] = a[x, z] + b[y, z]$
  - Alternating
    - $[x, x] = 0$
  - Satisfies the Jacobi identity
    - $[x, [y, z]] + [z, [x, y]] + [y, [z, x]] = 0$

# Root Systems

- A **root system** over some finite-dimensional Euclidean vector space  $V$  is a set of non-zero vectors (roots) which satisfy the following properties:
  - The roots span  $V$
  - The only scalar multiples of a root  $x$  appearing in the set are  $x$  and  $-x$ .
  - For every root, the set is closed under reflection through the hyperplane perpendicular to that root.
  - $V$  must be an inner-product space.
  - For roots  $\alpha, \beta$ ,  $\langle \alpha, \beta \rangle := 2(\beta, \alpha)/(\alpha, \alpha)$  is an integer.

# Examples of Root Systems

	
<p>Root system <math>A_1 \times A_1</math></p> 	<p>Root system <math>D_2</math></p> 
	
<p>Root system <math>A_2</math></p> 	<p>Root system <math>G_2</math></p> 

# Our Basis

$$h_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$e_{12} = \begin{bmatrix} 0 & 1 & 0 \\ \hline & & \end{bmatrix}$$

$$e_{21} = \begin{bmatrix} \hline & & \\ 1 & 0 & 0 \\ \hline & & \end{bmatrix}$$

$$h_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$e_{13} = \begin{bmatrix} 0 & 0 & 1 \\ \hline & & \end{bmatrix}$$

$$e_{31} = \begin{bmatrix} \hline & & \\ 1 & 0 & 0 \\ \hline & & \end{bmatrix}$$

$$e_{23} = \begin{bmatrix} \hline & & \\ 0 & 0 & 1 \\ \hline & & \end{bmatrix}$$

$$e_{32} = \begin{bmatrix} \hline & & \\ 0 & 1 & 0 \\ \hline & & \end{bmatrix}$$

# Resources

- Jon Cohen
- *Introduction to Lie Algebras* by Karin Erdmann and Mark J. Wildon
- *Introduction to Lie Algebras and Representation Theory* by James E. Humphreys
- ...Mostly Jon Cohen