Lie Algebras to Root Systems

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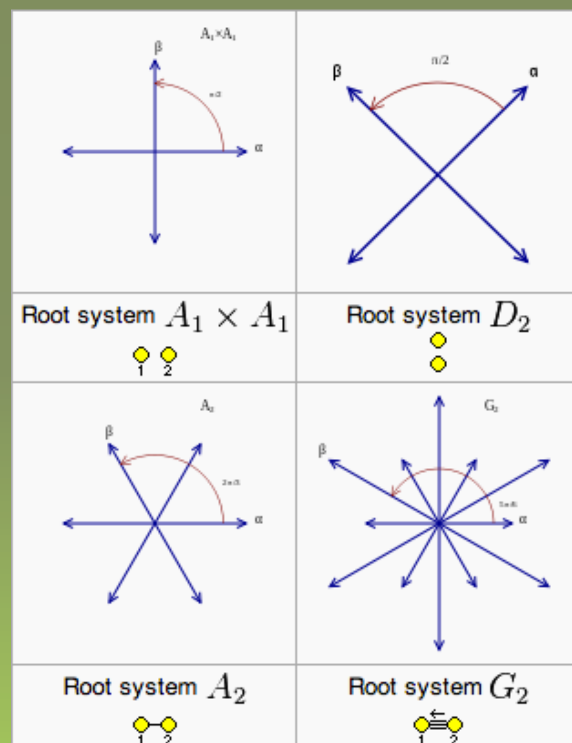
Lie Algebras

- A **Lie algebra** is a vector space over some field with some binary operation that satisfies the following axioms:
  - Bilinearity
    - $[ax + by, z] = a[x, z] + b[y, z]$
  - Alternating
    - $[x, x] = 0$
  - Satisfies the Jacobi identity
    - $[x, [y, z]] + [z, [x, y]] + [y, [z, x]] = 0$

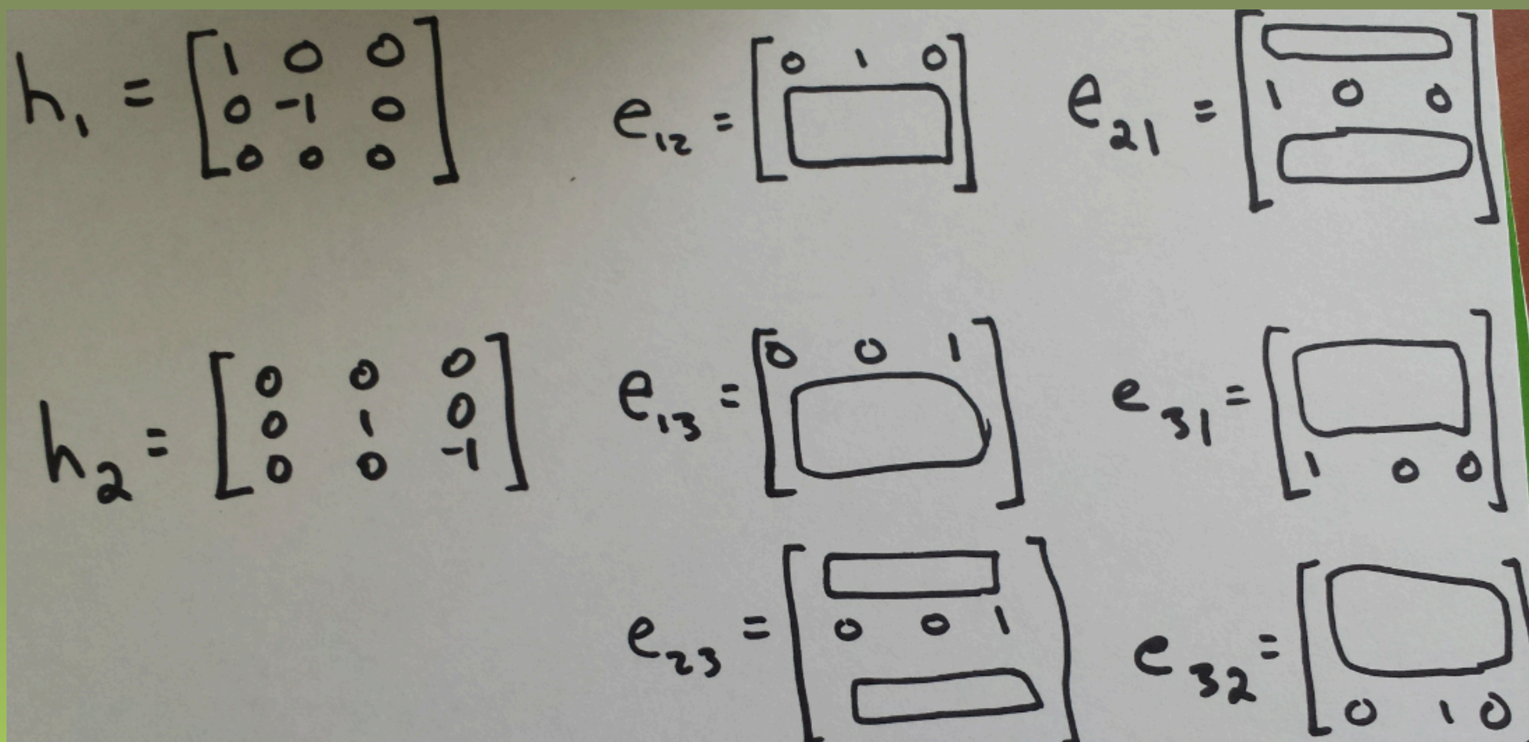
Root Systems

- A **root system** over some finite-dimensional Euclidean vector space *V* is a set of non-zero vectors (roots) which satisfy the following properties:
  - The roots span *V*
  - The only scalar multiples of a root *x* appearing in the set are *x* and –*x*.
  - For every root, the set is closed under reflection through the hyperplane perpendicular to that root.
  - *V* must be an inner-product space.
  - For roots α, β, $<\alpha, \beta> := 2(\beta, \alpha)/(\alpha, \alpha)$ is an integer.

Examples of Root Systems



Our Basis



Resources

- Jon Cohen
- *Introduction to Lie Algebras* by Karin Erdmann and Mark J. Wildon
- *Introduction to Lie Algebras and Representation Theory* by James E. Humphreys
- …Mostly Jon Cohen