Solving Disentanglement Puzzles with Hints from Topology

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Topological Space

Let X be a nonempty set and T a collection of subsets of X

- X is the underlying set
- T is the topology on the set X
- The members of T are called open sets

1. $X \in T$
2. $\emptyset \in T$
3. $If\ O_1, O_2,\ \ldots, O_n\ \in T, then\ O_1 \cap O_2 \cap \ldots \cap O_n\ \in T$
4. $If\ for\ each\ \alpha \in I, O_\alpha \in T, then\ \bigcup_{\alpha \in I} O_\alpha \in T$

The pair of objects (X,T) is called a topological space.

Example of a Topological Space

- Discrete Topology: Let X be an arbitrary set. Let T be the collection of all subsets of X, T = 2^X.

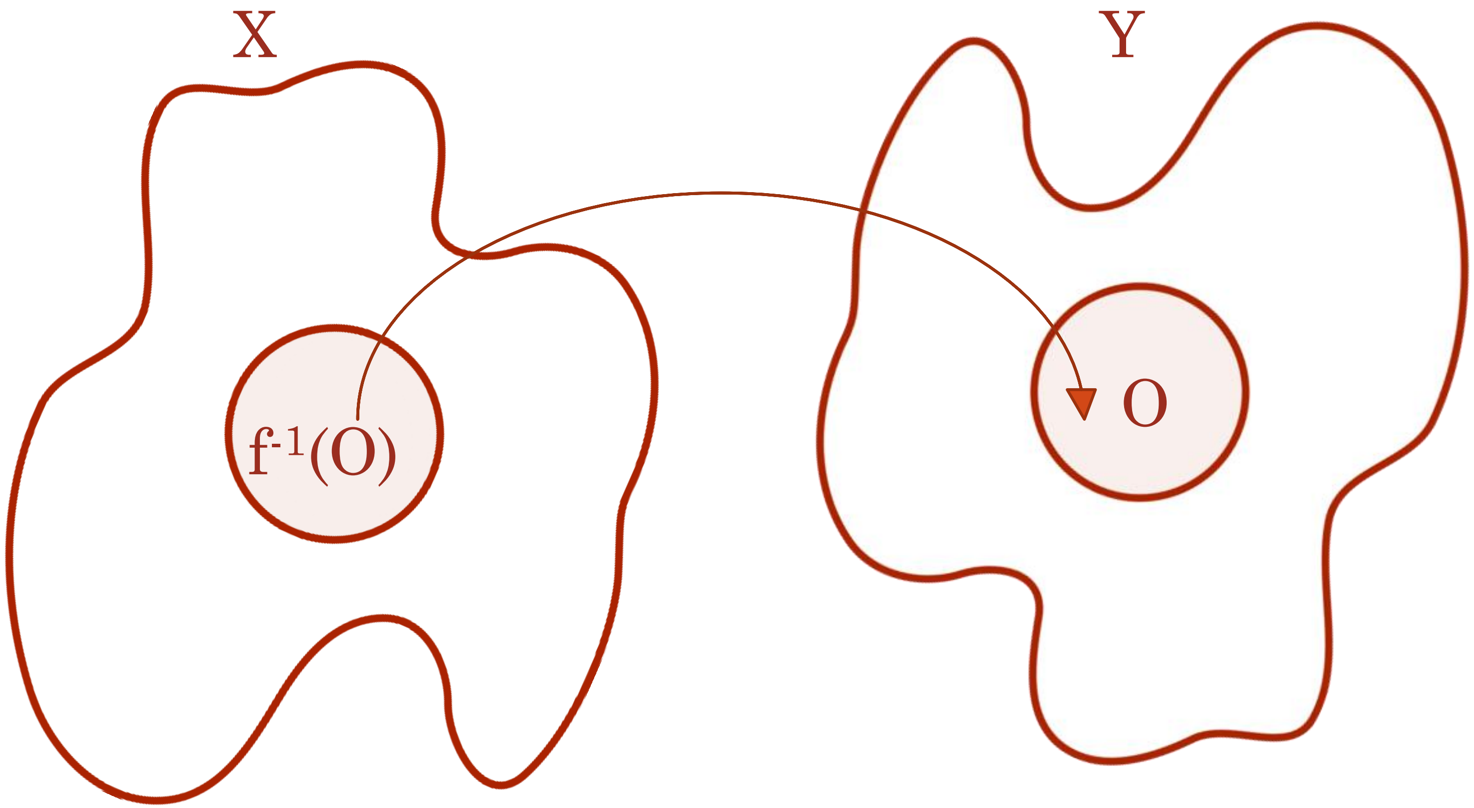
Let's check:

1. $X \in T$
2. $\emptyset \in T$
3. $If\ O_1, O_2,\ \ldots, O_n\ \in T, then\ O_1 \cap O_2 \cap \ldots \cap O_n\ \in T$
4. $If\ for\ each\ \alpha \in I, O_\alpha \in T, then\ \bigcup_{\alpha \in I} O_\alpha \in T$

Therefore (X, 2^X) is a topological space.

Continuity in a Topological Space

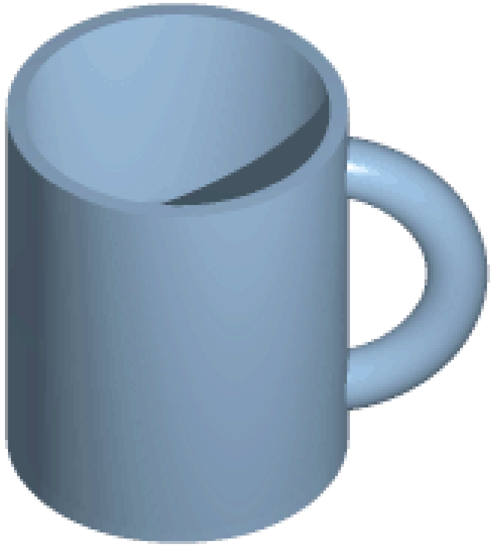
- A function f : (X,T) → (Y,T’) is said to be continuous if for each open set O in Y, $f^{-1}(O)$ is open in X.



Homeomorphism

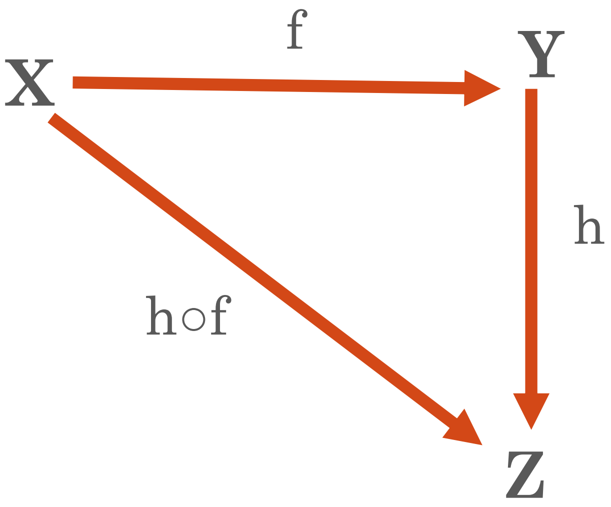
- Topological spaces (X,T) and (Y,T') are called homeomorphic if there exist continuous functions f: X → Y and g: Y → X with $f^{-1} = g$ and $g^{-1} = f$
- Theorem: A necessary and sufficient condition that two topological spaces (X,T) and (Y,T') be homeomorphic is that there exist a function f: X → Y such that:

1. f is one-to-one
2. f is onto
3. A subset O of X is open if and only if f(O) is open.



Example of Continuity and Homeomorphism

- Let f: (X,T) → (Y,T') be a homeomorphism. Let a third topological space (Z,T'') and a function h: (Y,T') → (Z,T'') be given. Prove that h is continuous if and only if $h \circ f$ is continuous.



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- f continuous by homeomorphism
- The composition of continuous functions is continuous
- As h is continuous $h \circ f$ must also be continuous

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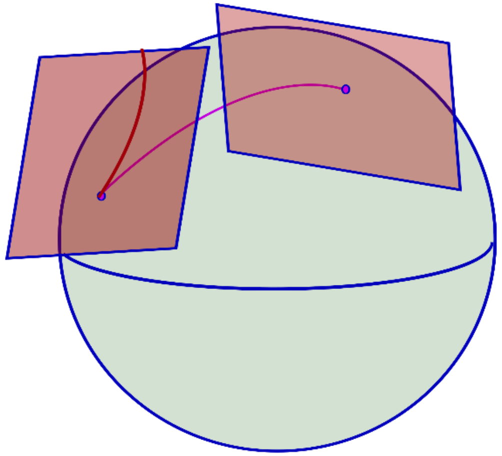
- $h(O) = (h \circ f)(f^{-1}(O))$
- $(h \circ f)$ is continuous and f^{-1} is continuous by homeomorphism
- The composition of continuous functions is continuous
- Therefore, h is continuous

Manifolds

- A topological space $M \subset R^m$ is a manifold if for every $x \in M$, an open set $O \subset M$ exists such that:

1. $x \in O$
2. O is homeomorphic to R^n
3. n is fixed for all $x \in M$ (dimension)



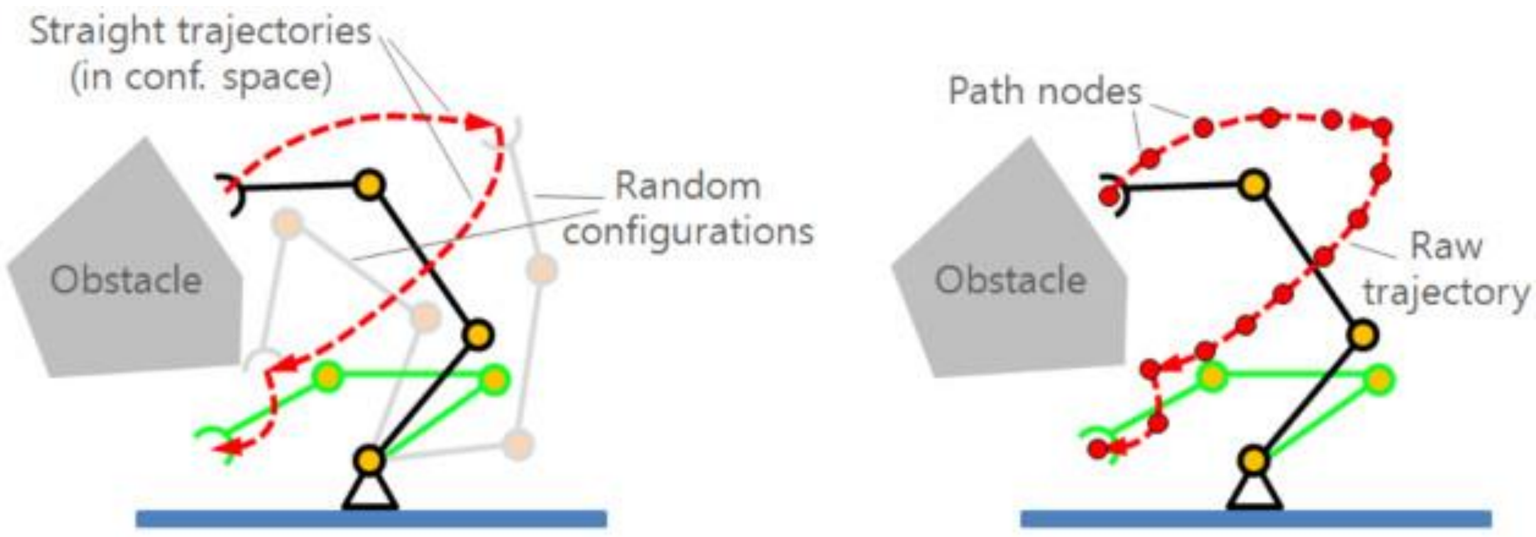
http://www.markushanke.net/manifolds-and-curvature

Mobius Strip

http://uofgts.com/Astro/cosmology-mobius.html

Configuration Space

- A configuration space is a manifold that comes from transformations.
- Can be thought of as degrees of freedom or all positions and orientations in space.
- SO(3) set of all rotations about the origin of $\mathbf{R}^3$.



http://www.coppeliarobotics.com/helpFiles/en/motionPlanningModule.htm

Disentanglement Puzzles

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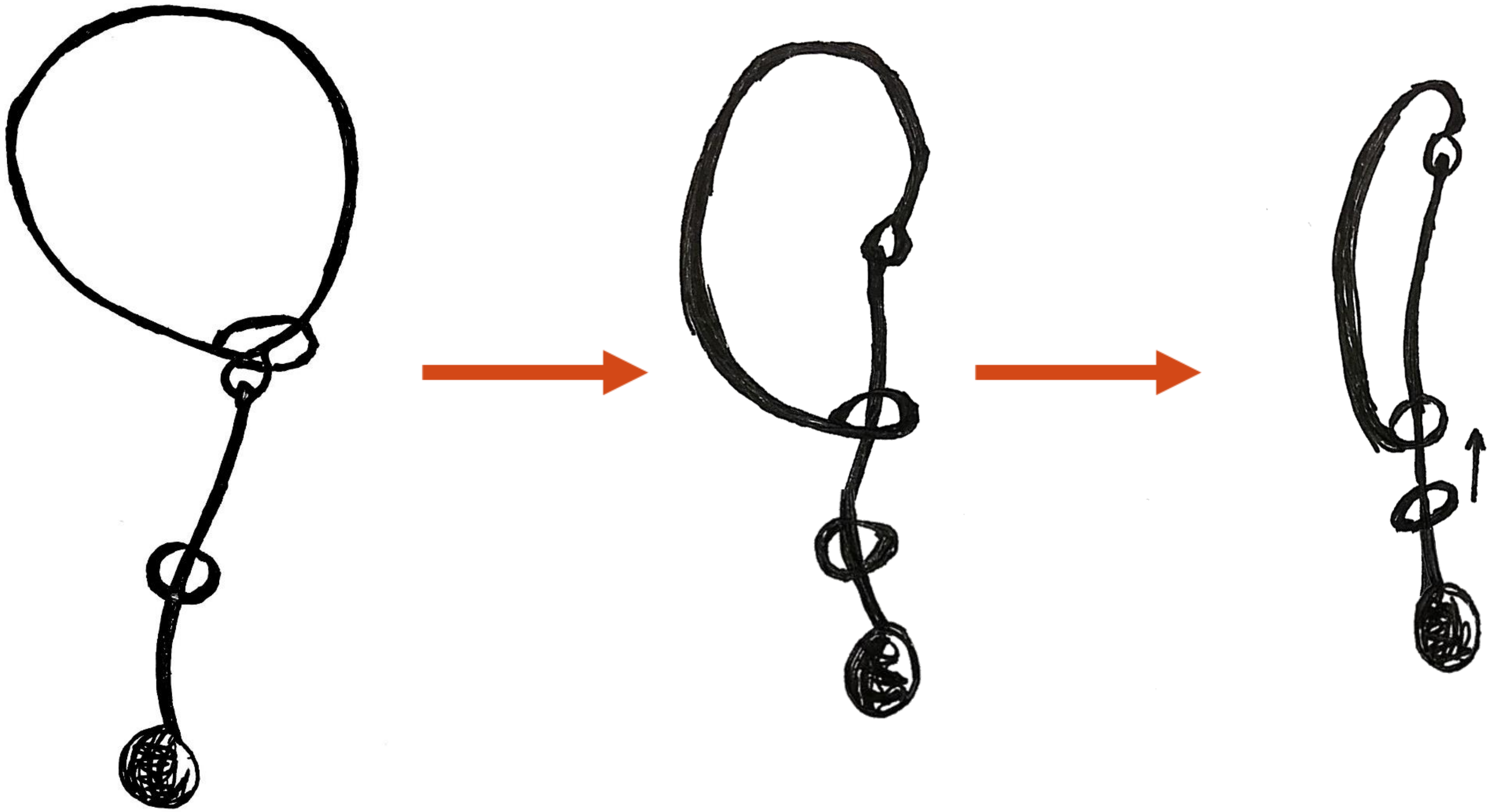
Hint at the Solution



Solution: Watch Closely!

https://youtu.be/L---R9LaJXo?t=10s

Sources

- *Introduction to Topology* 3rd Edition by Bert Mendelson
- Ch. 4: The Configuration Space from Steven M. LaValle's *Planning Algorithms*