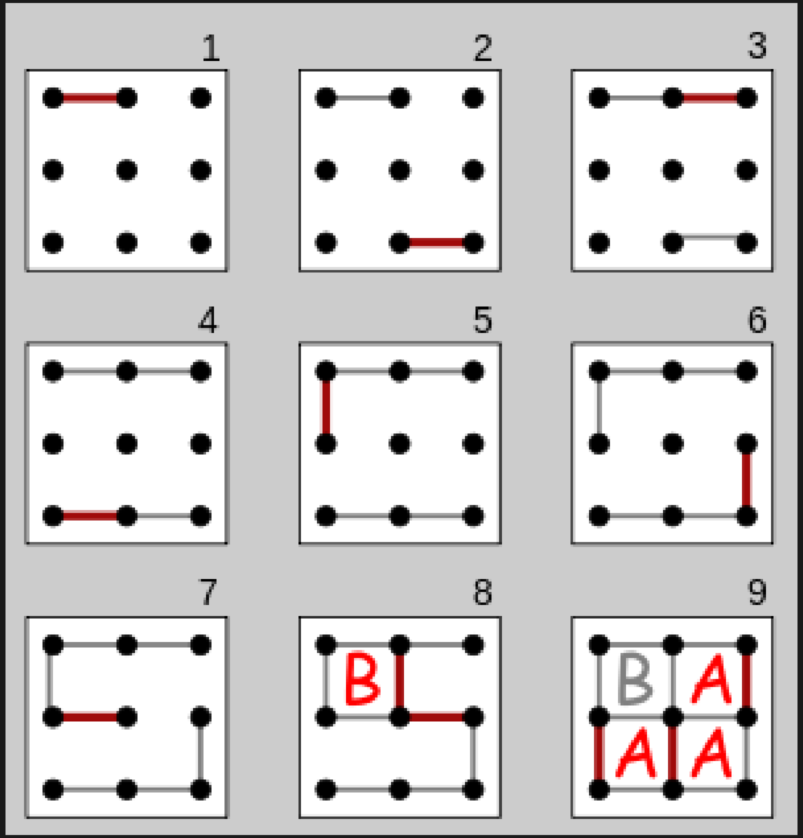
How to Beat a Small Child at Dots and Boxes

Alexa Tsintolas

Math and Games

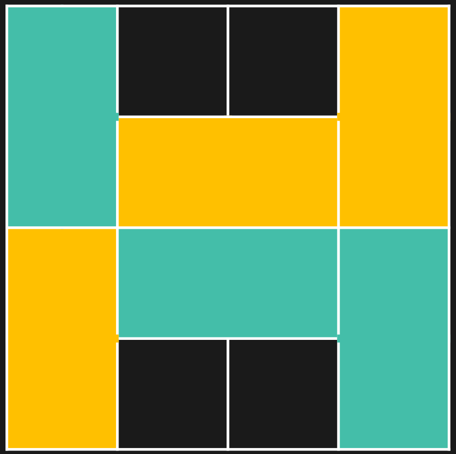
5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

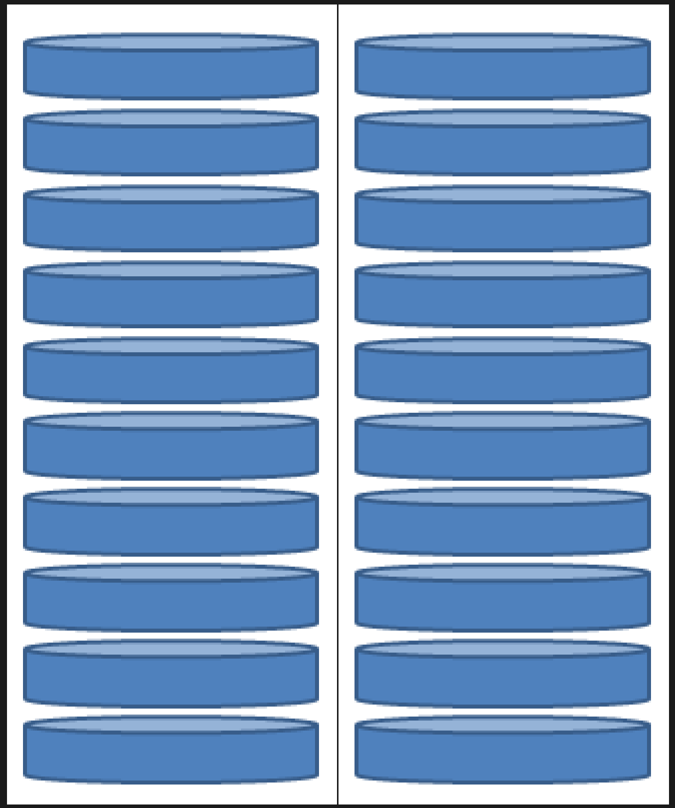


https://en.wikipedia.org/wiki/Dots_and_Boxes

Combinatorial Games

- 2 players take turns
- No random element devices like dice or spinners

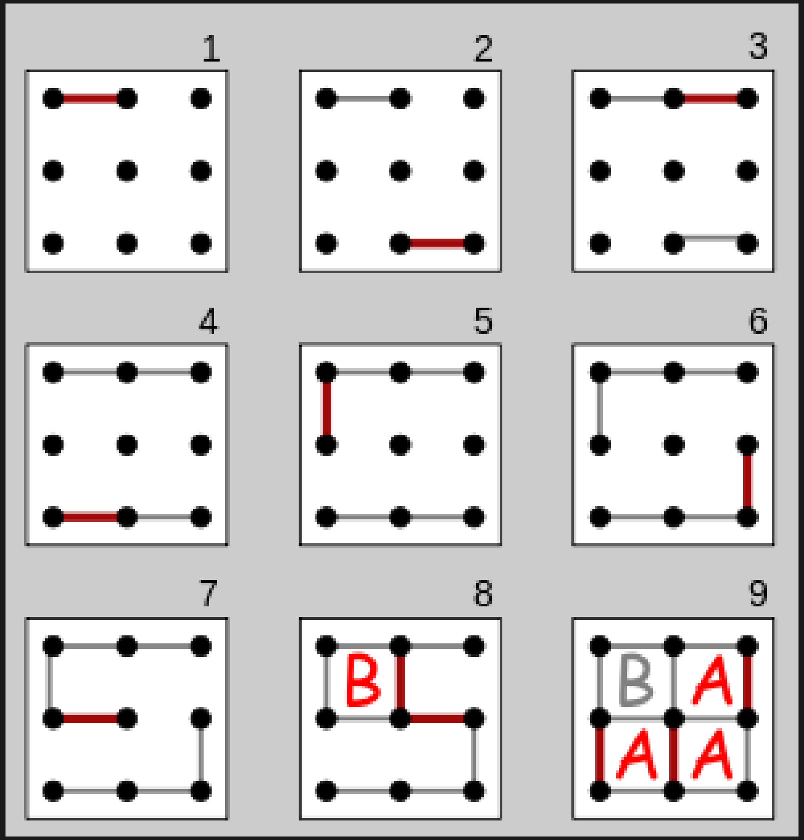




https://brilliant.org/wiki/nim/

Dots and Boxes

- 2 player paper and pencil game
- Array of dots
- Connect vertically/horizontally neighboring dots
- Goal is to make the most boxes
- After winning a box, the player goes again
- Game ends when no more boxes can be made



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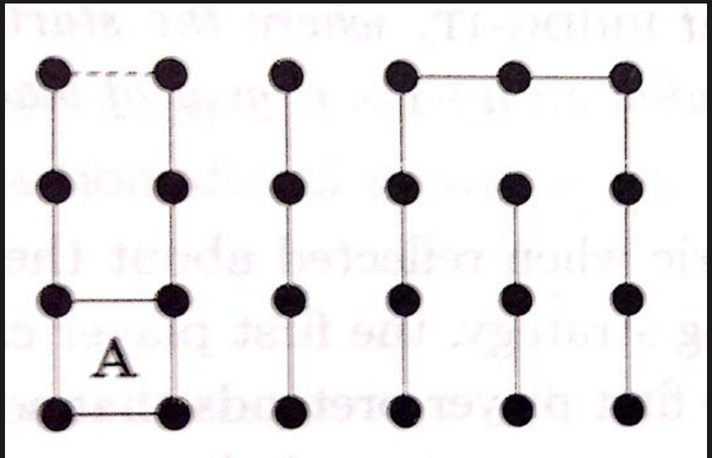
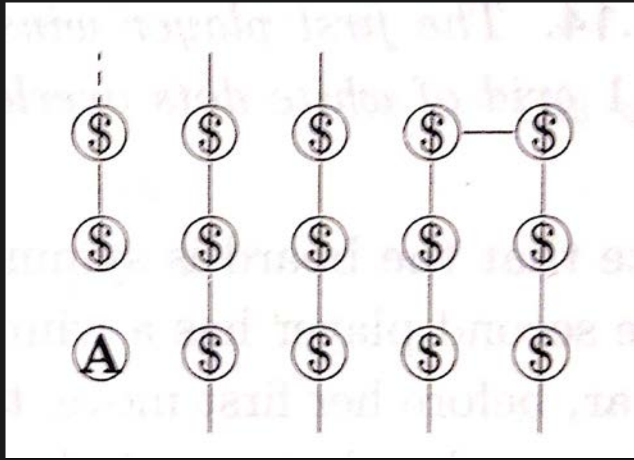
Strings and Coins

- Array of coins
- Cut vertical and horizontal strings
- Goal is to free the most coins
- After winning a coin, the player goes again
- Game ends when no more coins can be captured

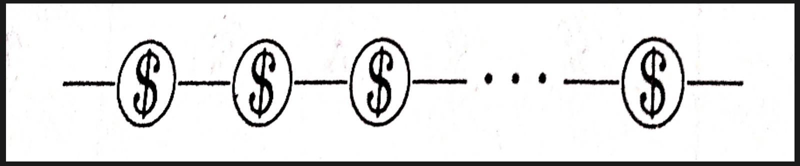




Dots and Boxes: Long Chains

- 3 or more boxes

- K ≥ 3 coins and exactly K+1 strings connected in a line



Long Chains Theorem

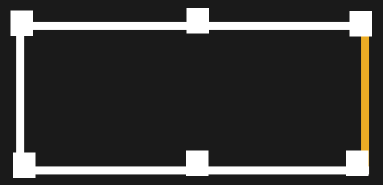
If a Dots and Boxes position is reduced to just long chains, player P can earn most of the remaining boxes, where

$$P \equiv M + C + B + D \ (mod\ 2)$$

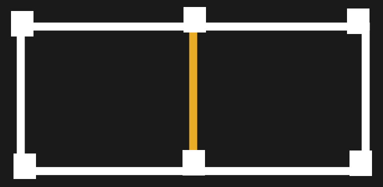
where the first player to move is player P = 1, and her opponent is player P = 2 (or, if you like P = 0).

The Double Cross

Double Dealing Move



Double Cross



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where the first player to move is player P = 1, and her opponent is player P = 2 (or, if you like P = 0).

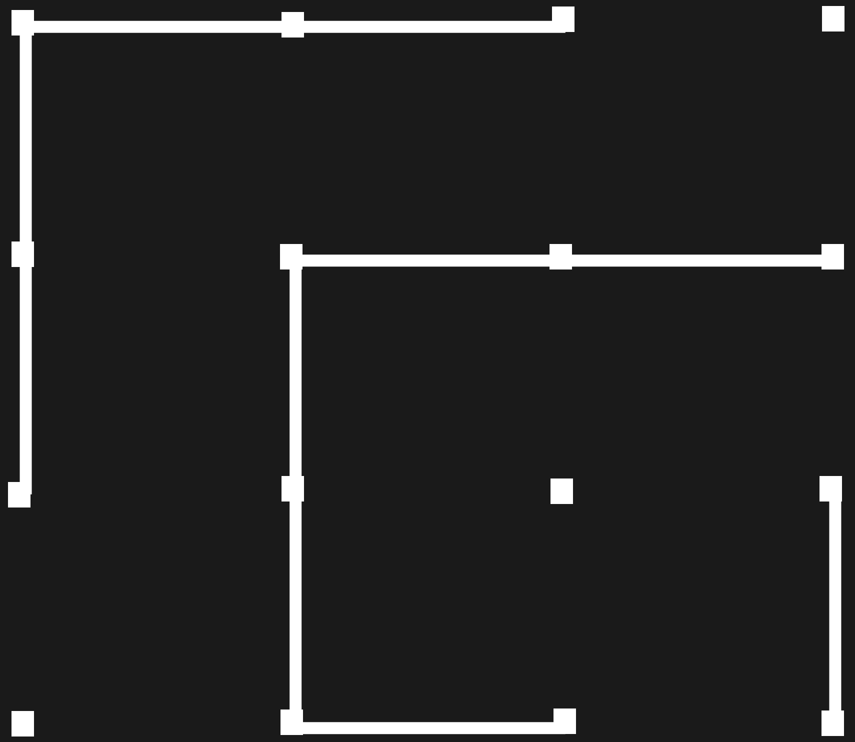
Example 1

$$P \equiv M + C + B + D \ (mod\ 2)$$

$$1 \equiv 24 + C + 9 + 0 \ (mod\ 2)$$

$$1 \equiv 33 + C \ (mod\ 2)$$

$$C \ even$$



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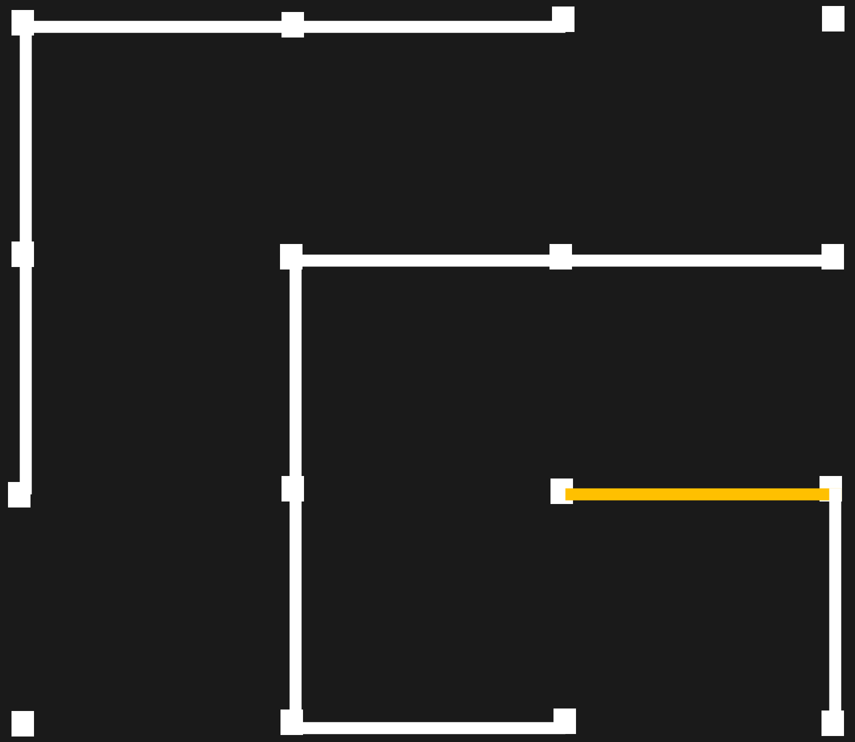


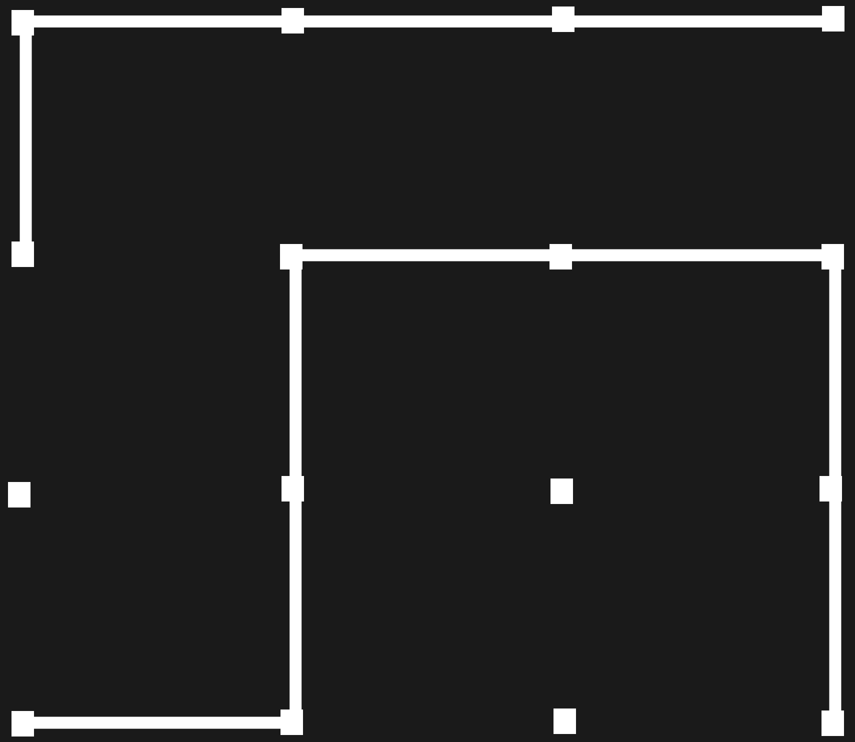
Example 2

$$P \equiv M + C + B + D \ (mod\ 2)$$

$$2 \equiv 24 + C + 9 + 0 \ (mod\ 2)$$

$$2 \equiv 33 + C \ (mod\ 2)$$

$$C \ odd$$



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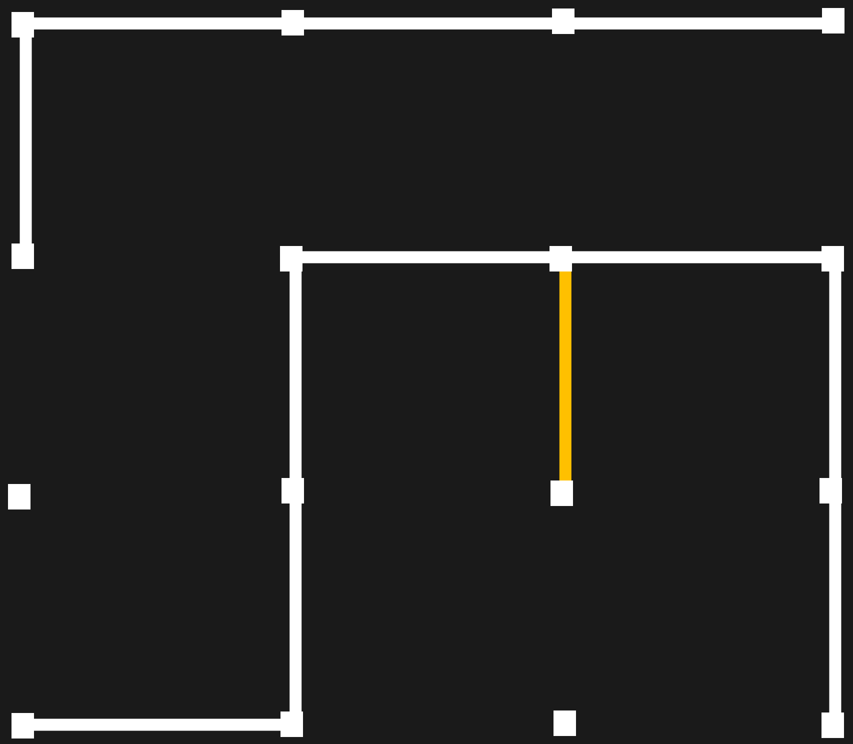


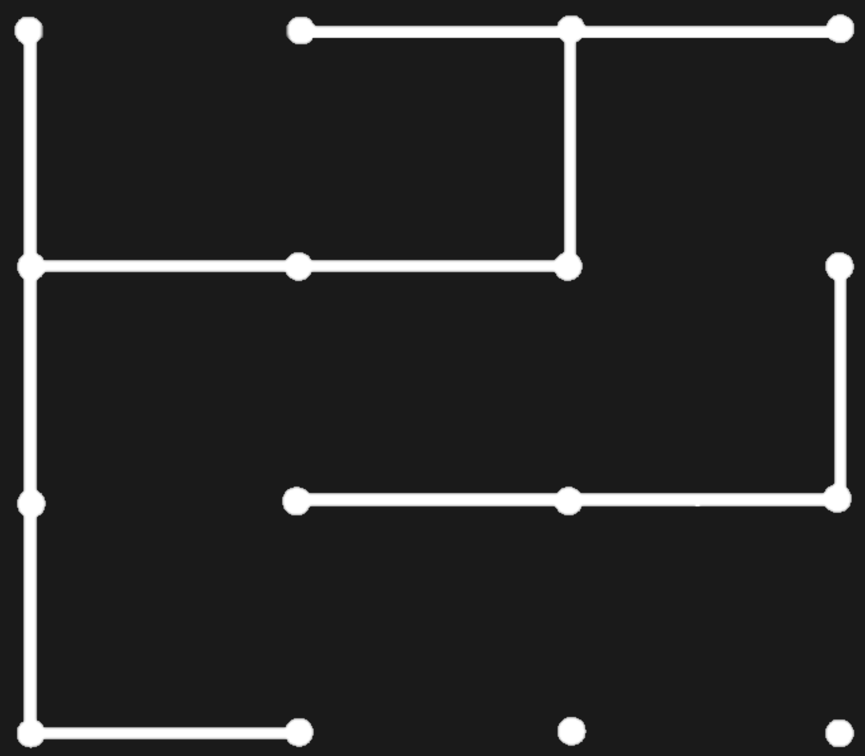
Example 3

$$P \equiv M + C + B + D \ (mod\ 2)$$

$$1 \equiv 24 + C + 9 + 0 \ (mod\ 2)$$

$$1 \equiv 33 + C \ (mod\ 2)$$

$$C\ even$$



Example 3

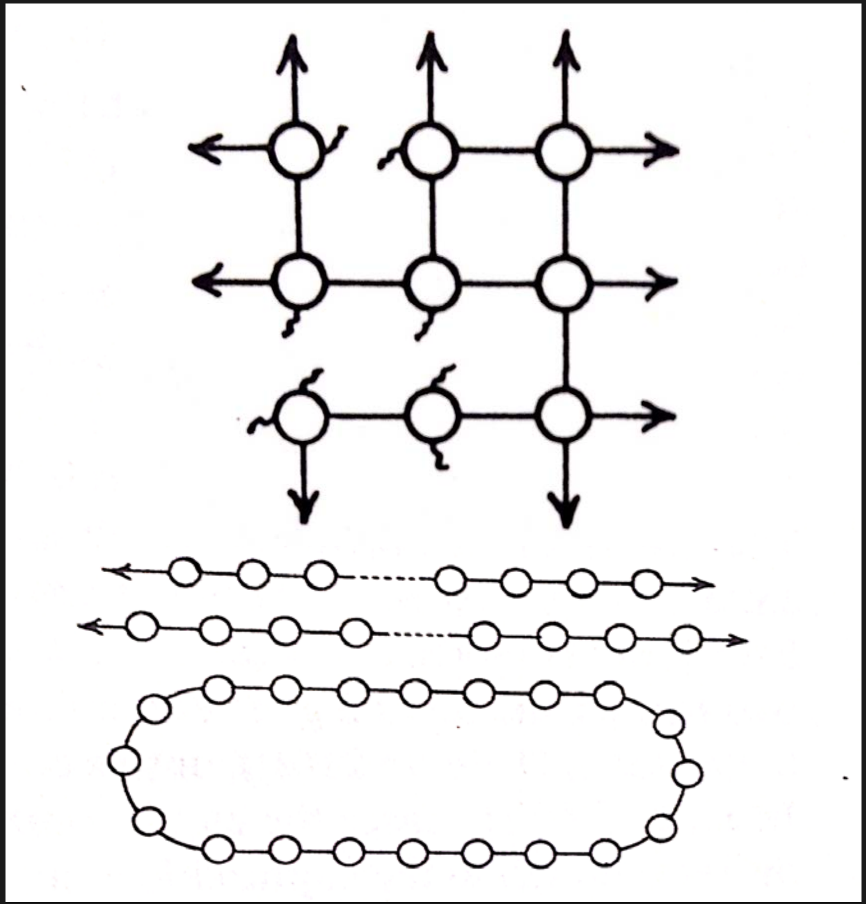
$P \equiv M + C + B + D \ (mod\ 2)$

$1 \equiv 24 + C + 9 + \mathbf{1} \ (mod\ 2)$

$1 \equiv 33 + C \ (mod\ 2)$

$C\ odd$

Nimstring



Resources

- Albert, Michael H., Richard J. Nowakowski, and David Wolfe. *Lessons in Play: An Introduction to Combinatorial Game Theory*. Wellesley: AK Peters, 2007. Print.
- Berlekamp, Elwyn R. *The Dots-and-boxes Game: Sophisticated Child's Play*. Natick: A.K. Peters, 2000. Print.
- Rob Maschal