Group Theoretic Approach to the Helicopter Cube

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Rubik's Cube

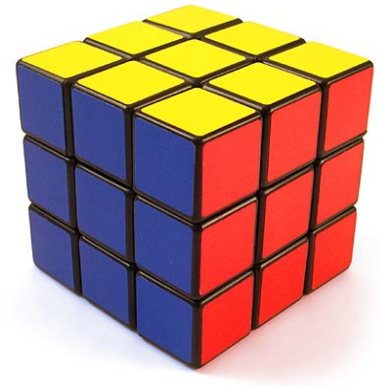


Figure: Picture from thinkgeek.com

The movements of the Rubik's Cube form a group!

- The group is a subgroup of the symmetric group S_{48}.
- The size of the group is 43,252,003,274,489,856,000.

Helicopter Cube

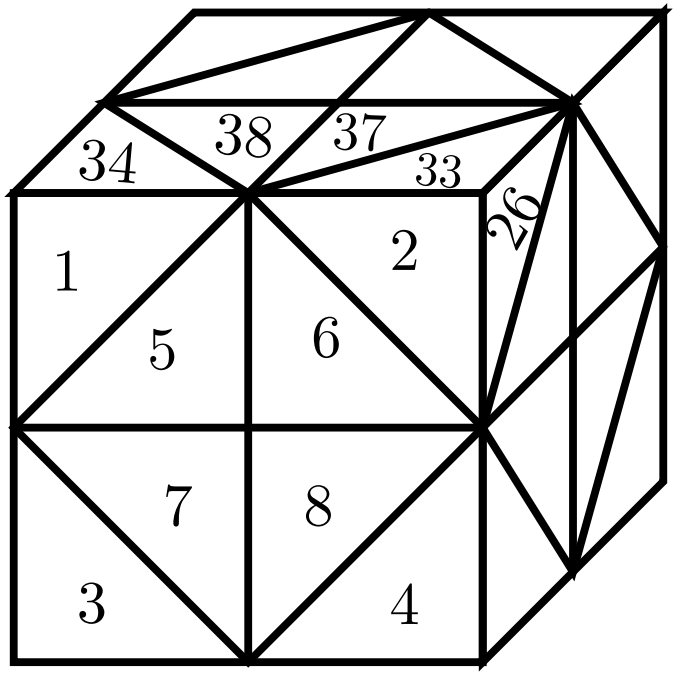
For this project, we wanted to work with the Helicopter cube.

Figure: Photo found on kostkirubika.pl

Where we study this puzzle similarly to the Rubik's Cube.

Generator Example

Moving one of the edges is one of the known generators



One example would be

$$Generator = (1, 33)(2, 34)(5, 37)(6, 38)(9, 26);$$

- The collection of all these edge moves **do not** generate the whole group.

GAP

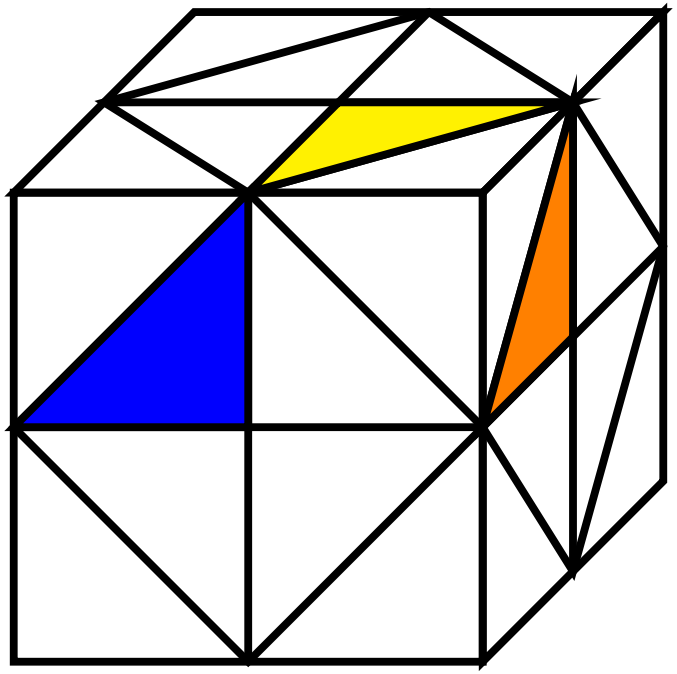
We used a program, GAP, where we inputted all the possible actions that generate the whole the group.

- We added all the actions on the cube into GAP.
- We can ask GAP questions about the group, such as the size, orbits, or if a certain action is in the group.

The Extra Moves

There exists a strange move

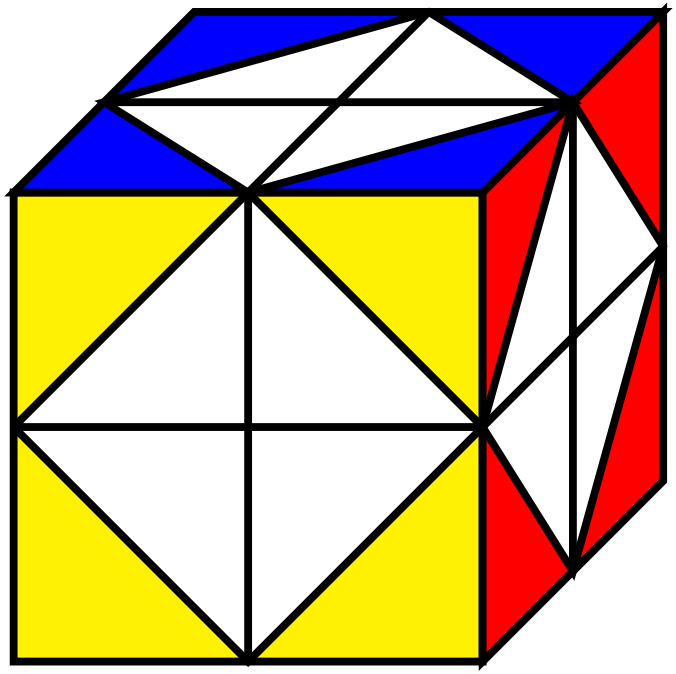
- With just the edge moves, the group size is 11,848,661,611,315,200,000.
  - Compared to the Rubik's Cube, which is 43,252,003,274,489,856,000.
- With the new moves, the group size goes up to 27,355,520,396,546,388,222,227,251,200,000!

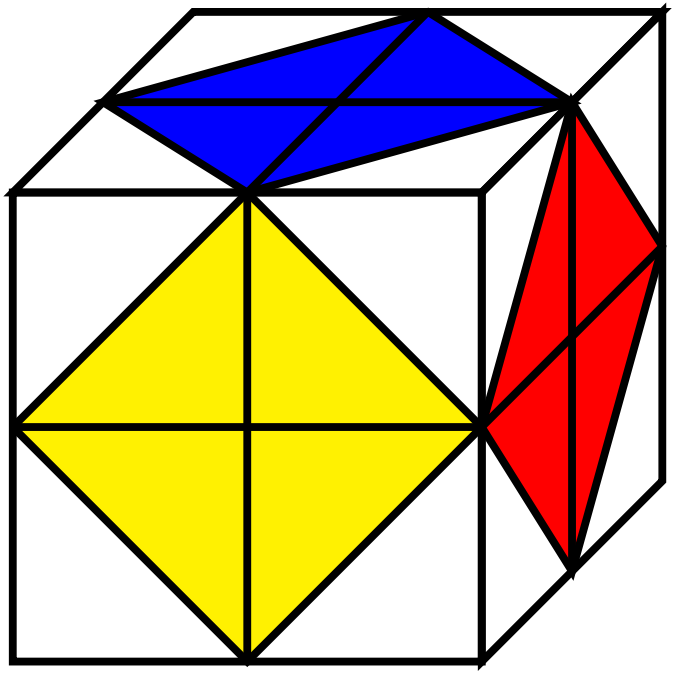


Orbits

There are two main orbits that make up the Helicopter Cube, similar to the Rubik's Cube.

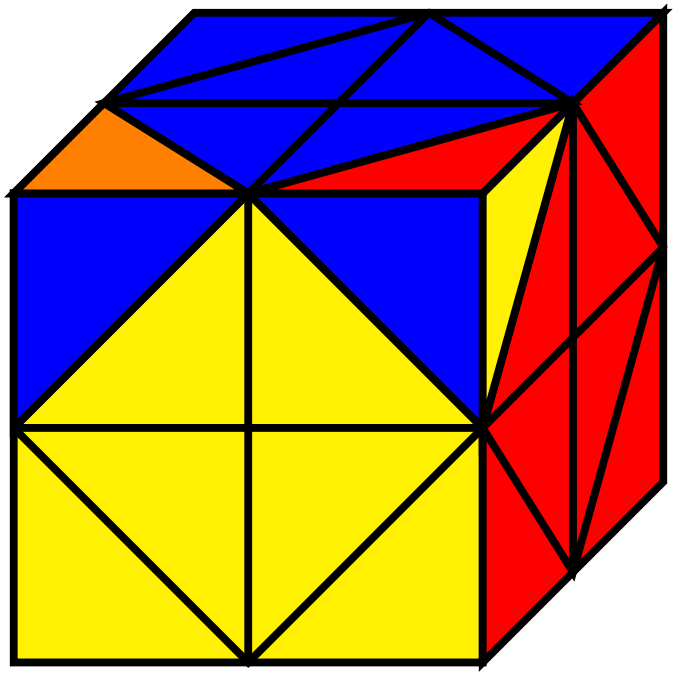
- The corner pieces fall in one orbit
- The center pieces fall into a different orbit.





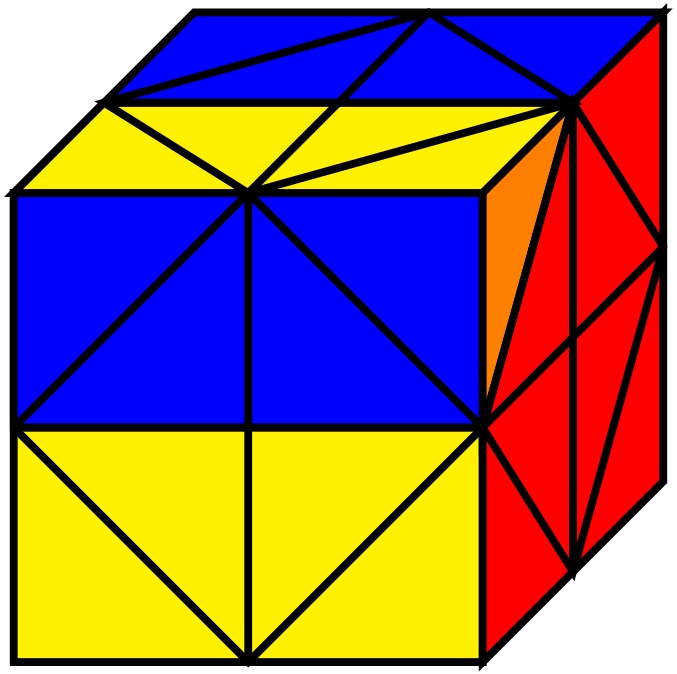
The Normal Subgroups

We can think of some of the normal subgroups that make up the whole group. Such a group would keep the positions the same, but change the orientation of the pieces. Such a move would give us:



Generators

We considered the possibility of generating an edge move with a combination of other movements. Such a combination would create:



Finding such a combination would allow us to conclude that there are fewer than 24 generators of the whole group.
In fact, with the help of GAP, we found that there are 14 generators.

Conclusion

The Rubik's Cube group and the Helicopter Cube group are similar in many ways, but the Helicopter Cube's unique movements make the group a little more complicated.
We can use GAP to learn more about, not only the Helicopter Cube, but other puzzles as well.